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Resonant excitation of off-channel localized impurity modes by a photonic crystal waveguide

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Abstract
A theoretical treatment is presented of the electromagnetic transmission properties of a photonic crystal waveguide that interacts with off-channel dielectric impurities, impurity clusters, neighbouring waveguides, or neighbouring waveguide networks. The photonic crystal studied is a two-dimensional square lattice array of parallel axis dielectric cylinders (formed of linear dielectric medium) in vacuum, and impurities and waveguides are created by cylinder replacement. The waveguide whose transmission is calculated is formed from linear dielectric media, but the off-channel features that it interacts with may be formed from either linear or Kerr nonlinear dielectric media. The off-channel features may also be composed of dissipative or amplifying media. Waveguide transmission resonances, associated with resonant scattering from electromagnetic modes on the off-channel features, are found. Modes present on off-channel features formed from linear dielectric media include both propagating and localized modes. Modes present on off-channel features formed from Kerr nonlinear dielectric media include simple localized and intrinsic localized (soliton-like) modes.

1. Introduction

The use of photonic crystals to control the flow of electromagnetic energy through space has been a topic of increasing interest during the last decade [1–4]. Photonic crystals are periodic dielectric arrays exhibiting a frequency band structure for the propagation of electromagnetic waves. This band structure is similar to the energy band structure of electrons in insulating, metallic, and semiconducting crystals arising from the periodic potential of the positive ions. Photonic crystals can be designed to be periodic in one dimension (layered optical media), two dimensions, or three dimensions, with corresponding band structure restrictions. The series of electromagnetic frequency stop and pass bands of photonic crystals allow for their use to filter, confine, or steer the flow of electromagnetic energy. These properties have led to a variety of applications and suggestions for applications.

In addition to their properties as pure periodic systems, photonic crystals containing isolated impurities have potentially useful applications [1–4]. Dielectric impurities introduced into photonic crystals bind impurity electromagnetic modes similar to the bound donor and acceptor electron and hole modes found in doped semiconductors. Dielectric impurity modes in photonic crystals, like semiconductor impurity modes, occur at stop band frequencies and are localized about impurity sites. Such localized electromagnetic modes can function as Fabry–Perot resonators in the design of lasers. They also offer a means to create high Q electromagnetic resonant cavities not accessible by other technologies.

Ideas developed in the treatment of single-site impurities are easily extended to impurity arrays [1–7]. For example, a linear array of single-site impurities in a two- or three-dimensional photonic crystal can function as a waveguide [1–11]. The linear array of single-site impurities acts as a waveguide channel that binds electromagnetic (waveguide) modes to propagate only along the length of the channel. The medium in the channel is chosen such that the waveguide modes occur at frequencies in the stop gap of the bulk photonic crystal. The electromagnetic energy in such modes moves along the channel but cannot radiate into the bulk of the photonic crystal. In this paper we focus on the transmission properties of waveguides formed as linear arrays of single-site impurities in a two-dimensional photonic crystal that is a square lattice array of infinite dielectric cylinders. Only the properties of electromagnetic modes propagating in the plane of the square lattice are of interest. We extend previous work by us [5–11] on this system to study the effects of off-channel impurities [3, 4, 12–15], impurity clusters [4, 16], and neighbouring waveguide networks [17–19] on the transmission of electromagnetic energy in a straight infinitely long waveguide formed from linear dielectric media. Off-channel impurities formed from both linear and nonlinear dielectric media are considered. The interest in this topic comes from the fact that in photonic crystals containing systems of multiple waveguides and impurities interactions will exist between these components. Such interactions modify waveguide transmission characteristics and must be factored into design applications.

Two general treatments of waveguides in photonic crystals have been given. In the first approach waveguides are formed in the photonic crystal by removing a channel (a row or a number of parallel rows) of sites from the photonic crystal. The modes in the resulting system are then studied by computer simulation [1–4]. In this paper our interest is in a second approach to the waveguide problem [5–11]. A waveguide in a two-dimensional photonic crystal is created by replacing an array of dielectric cylinders forming the waveguide channel by cylinders with different dielectric properties from those of the bulk two-dimensional photonic crystal. (In this scheme the cylinders that are replaced need not be nearest-neighbour cylinders in the photonic crystal. For example, a waveguide channel can be made by replacing every fifth cylinder in a row by a cylinder with different dielectric properties from those of the bulk two-dimensional photonic crystal.) The exact solutions for the electromagnetic modes in the system are obtained from an integral equation eigenvalue problem. Under proper conditions the integral equation eigenvalue problem reduces to a simple set of difference equations. The resulting difference equations yield exact analytical solutions for the fields of the waveguide modes. In the next section an outline of the development of the difference equation approach is given. The details of this development can be found in [6, 8].

2. Difference equation formulation of waveguide modes

We consider a photonic crystal that is a square lattice array of parallel axis infinite dielectric cylinders in vacuum [6, 8]. The cylinders are made of linear dielectric medium and are of circular cross section. Impurities and waveguides are introduced into the photonic crystal by cylinder replacement.

The equations describing a waveguide created by cylinder replacement in a two-dimensional photonic crystal are simple in form [5, 6, 8]. Let $\delta\epsilon(\vec{r}_\parallel)$ represent the change, from the bulk of the photonic crystal, of the dielectric constant in the waveguide channel. For waves propagating in the plane of the two-dimensional photonic crystal lattice with electric field polarized along the axes of the dielectric cylinders, the field amplitude as a function of position is a solution of [5, 6, 8]

$$E(\vec{r}_\parallel) = \int \mathrm{d}^2 r'_\parallel \, G(\vec{r}_\parallel, \vec{r'}_\parallel|\omega)\delta\epsilon(\vec{r'}_\parallel)\frac{\omega^2}{c^2}E(\vec{r'}_\parallel). \tag{1}$$

Here $\vec{r}_\parallel = x\hat{i} + y\hat{j}$ is in the plane of the photonic crystal lattice, ω is the frequency of the mode (for a waveguide mode, ω must be in a stop band of the photonic crystal), $G(\vec{r}_\parallel, \vec{r'}_\parallel|\omega)$ is the Green function of the Helmholtz wave equation of the photonic crystal without waveguides or impurities, and the electric field $E(\vec{r}_\parallel)\mathrm{e}^{-\mathrm{i}\omega t}$ is independent of z. The function $\delta\epsilon(\vec{r}_\parallel)$ is a type of step function in space; i.e., it is only non-zero and equal to $\delta\epsilon_{00}$ at channel sites or portions of channel sites and is otherwise zero. Consequently, equation (1) is a Fredholm integral eigenvalue problem. The eigenvalues, $\delta\epsilon_{00}$, and corresponding eigenmodes of the waveguide, $E(\vec{r}_\parallel)$, are determined as functions of the waveguide mode frequency ω.

Under certain conditions equation (1) reduces to a set of difference equations relating the field amplitude at a given waveguide channel site to the field amplitude at its two neighbouring sites [5–7]. This occurs, for example, when the field changes slowly over each separate region of non-zero $\delta\epsilon_{00}$ along the waveguide channel. In this limit $E(\vec{r}_\parallel)$ is constant over the replacement material at a single channel cylinder so that equation (1) becomes an algebraic (difference) equation. The form of the resulting difference equation obtained from equation (1) is then [6, 7]

$$E_{n,0} = \gamma[aE_{n,0} + b(E_{n+1,0} + E_{n-1,0})]. \tag{2}$$

In equation (2) the waveguide is along the x-axis so that $(n, 0)$, for n an integer, labels channel sites in the x–y-plane. The coefficients γ, a, and b are obtained from equation (1). The factor γ is proportional to the dielectric contrast $\delta\epsilon_{00}$. The couplings a and b are related, respectively, to averages of the Green functions over the same replacement channel site and between closest neighbouring replacement channel sites. The couplings a and b depend on the mode frequency ω and the geometric and dielectric properties of the photonic crystal in the absence of impurities and waveguides. (The reader is referred to [6] for a detailed discussion of $\delta\epsilon_{00}$, γ, a, and b.) By changing the separation between nearest-neighbouring replacement sites along the channel, the number of fields coupled to E_n in equation (2) can be varied [6, 10]. For simplicity, we study below a case in which E_n only couples to E_{n+1} and E_{n-1}, i.e., not to E_{n+2} and E_{n-2}, etc.

3. Off-channel single sites and finite clusters

In this section the effects on waveguide transmission of interactions with off-channel single sites and clusters of impurity sites are considered. Resonant transmission anomalies of the waveguide are found to be associated with bound states that would exist on the single-site impurities or clusters of impurity sites when they are located in the bulk of the photonic crystal. It is shown that off-channel impurities can be designed with specified density of states. This flexibility allow us to engineer the transmission characteristics of the waveguide in the waveguide–impurity system.

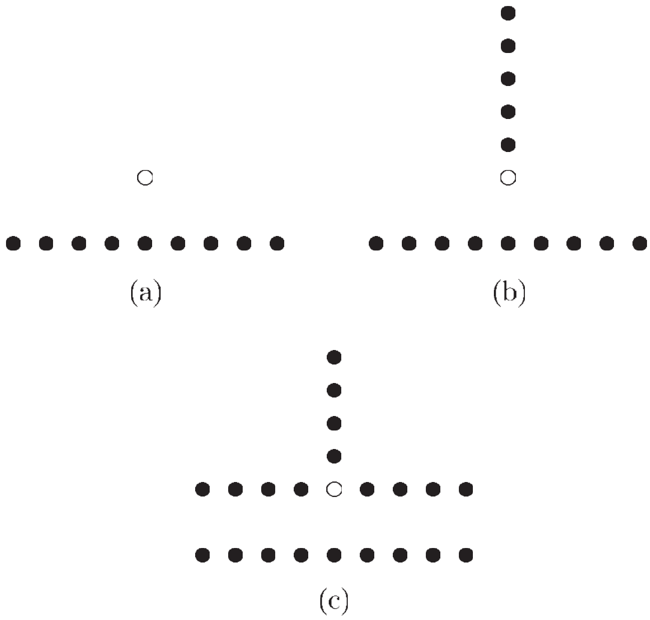

Figure 1. Schematic representation of some waveguide and off-channel impurity geometries considered in the paper. These are the following. (a) A straight infinitely long waveguide (closed circles) interacting with a single off-channel site (open circle). (b) A straight infinitely long waveguide (horizontal closed circles) interacting with a single off-channel site (open circle) that is at the lower end of a semi-infinite waveguide (vertical closed circles). (c) A straight infinitely long waveguide (lower horizontal closed circles) interacting with an impurity site (open circle) that is at the junction of three semi-infinite waveguides (remaining closed circles). In these diagrams the closed circles are all formed of the same linear dielectric medium and the open circles are of a different type of replacement medium that may or may not be Kerr nonlinear.

3.1. Single site

For the case of a single off-channel site consider a straight waveguide formed from linear dielectric media. (See figure 1(a) for a schematic representation of the waveguide channel and off-channel site impurity.) The waveguide channel is along the x-axis at sites labelled $(n, 0)$ where n are integers. The off-channel site is located on the y-axis at $(0, p)$ for $p > 0$ an integer and is composed of either linear or nonlinear dielectric media that may have a dissipative or amplifying component. Dissipative components simulate off-channel sites used to siphon energy from the waveguide. Amplifying components simulate enhancement effects similar to those found in Er doped optical fibres [20].

The set of difference equations describing this system is [6, 8]

$$E_{n,0} = \gamma[aE_{n,0} + b(E_{n+1,0} + E_{n-1,0})] \tag{3}$$

where $|n| \geqslant 1$,

$$E_{0,0} = \gamma[aE_{0,0} + b(E_{1,0} + E_{-1,0})] + cf_\epsilon(E_{0,p}), \tag{4}$$

and

$$E_{0,p} = af_\epsilon(E_{0,p}) + \gamma cE_{0,0}. \tag{5}$$

In equation (5) c is a coupling between the $(0, 0)$ site of the waveguide channel, and the off-channel site at $(0, p)$. The function $f_\epsilon(E)$ defined by

$$f_\epsilon(E) = \gamma'[1 + \lambda|E|^2]E + \mathrm{i}\epsilon E \tag{6}$$

accounts for the interaction of the off-channel site with the waveguide. The first term in the brackets in equation (6) is a linear dielectric interaction, the second term in the brackets is a Kerr nonlinearity, and the term on the far right models the effects of dielectric losses or amplification at the off-channel site. (See [8] for details of the simple generalization of equations (2) to Kerr nonlinear media.) The cases $\epsilon > 0$ and $\epsilon < 0$ represent, respectively, the system with dielectric losses and amplification. It is assumed that the strongest coupling between the off-channel site and the waveguide channel is between the $(0, p)$ and the $(0, 0)$ sites. Only this coupling is retained in the calculations.

The reflection and transmission coefficients for propagating waveguide modes incident onto the channel coupling between $(0, 0)$ and $(0, p)$ from left infinity (i.e., $(n, 0) = (\infty, 0)$) are calculated exactly from equations (3)–(6). For $n < 0$, the waveguide modes are of the form $E_{n,0} = u\mathrm{e}^{\mathrm{i}kn} + r\mathrm{e}^{-\mathrm{i}kn}$ where u and r are the amplitudes of the incident and reflected waves, respectively. For $n > 0$, the waveguide modes are of the form $E_{n,0} = t\mathrm{e}^{\mathrm{i}kn}$ where t is the transmission amplitude. Substituting these forms into equations (3)–(6) gives u and r in terms of t. The solvability condition for equations (3)–(6) gives the dispersion relation $\omega(k)$ of the waveguide modes as a solution of $1 = \gamma[a(\omega) + 2b(\omega)\cos k]$. (Here $a(\omega)$ and $b(\omega)$ are frequency dependent parameters from the bulk photonic crystal.) The reflection and transmission coefficients are given by $R = |r/u|^2$ and $T = |t/u|^2$ so that for the transmission

$$T = \frac{4\sin^2 k}{[2\sin k + \frac{cs_1}{b}(1 + 2\frac{b}{a}\cos k)]^2 + [\frac{c}{b}(1 + 2\frac{b}{a}\cos k)(s_0 - \gamma c)]^2}. \quad (7)$$

In equation (7) $s = s_0 + \mathrm{i}s_1$ is a complex solution of the cubic-like equation $\lambda t^2|s|^2 s + (1 - \frac{1-\mathrm{i}a\epsilon}{\gamma' a})s + \gamma c/(\gamma' a) = 0$, and the field amplitude in the off-channel site is $E_{0,p} = st$.

We first consider the case in which the off-channel site is made from linear dielectric media, i.e., $\lambda = 0$. For this case $s = -\gamma c/[\gamma' a - (1 - \mathrm{i}a\epsilon)]$ so that if $|a\epsilon| \ll 1$ a maximum occurs in s at $\gamma' a \approx 1$. This maximum comes from the presence of a bound state of the single-site impurity at $\gamma' a = 1$ [7]. The waveguide reflection and transmission coefficients exhibit anomalies associated with resonant scattering from this bound state.

In figure 2(a) results are shown for an example linear system. The cases $\epsilon = 0$, $\epsilon < 0$ (amplifying medium), and $\epsilon > 0$ (lossy medium) are treated. For these plots we have taken $b/a = 0.0869$ from the photonic crystal waveguide system studied in [6, 7, 10, 11] and $k = 2.5$ as a representative wavenumber. The coupling $c/a = 0.02$ and the $a\epsilon = \pm 0.002$ weak amplification and dissipation terms are chosen to give typical plots, illustrating the effects of these perturbations on the system. The results for $\epsilon = 0$ exhibit a transmission minimum and are qualitatively similar to the computer simulation results of Noda *et al* [3] for a different version of the off-channel impurity problem. (Our results are for a model that has an analytical solution, while the model treated by Noda was accessible only to computer simulation methods and did not treat dissipative and amplifying media.) The minimum arises as a resonance with a bound state of the single-site impurity when located in the bulk photonic crystal. Away form the resonance the off-channel site has little effect on the waveguide transmission. The $\epsilon < 0$ curve shows the effects on the waveguide transmission of an amplifying medium in the off-channel site. The transmitted energy in this case can be enhanced over that originally incident on the scattering site, and from equation (7) we find that $T > 1$ at resonance for $\epsilon < -\frac{(\gamma' c)^2}{4b\sin k}$ where $|\epsilon| \ll \gamma'$, $|\frac{ab}{c^2}\sin k|$. (Note that the dependence here of ϵ on $1/b$ is not a problem so long as the wavevector and group velocity of the waveguide modes are parallel.) For general $\epsilon < 0$, $T + R > 1$ as the amplifying medium is an energy source. The $\epsilon > 0$ curve shows the effects on the waveguide transmission of dissipative loss in the off-channel site. For $\epsilon > 0$ the transmission is slightly enhanced at resonance, but $T + R < 1$ as the dissipative medium gives an energy loss.

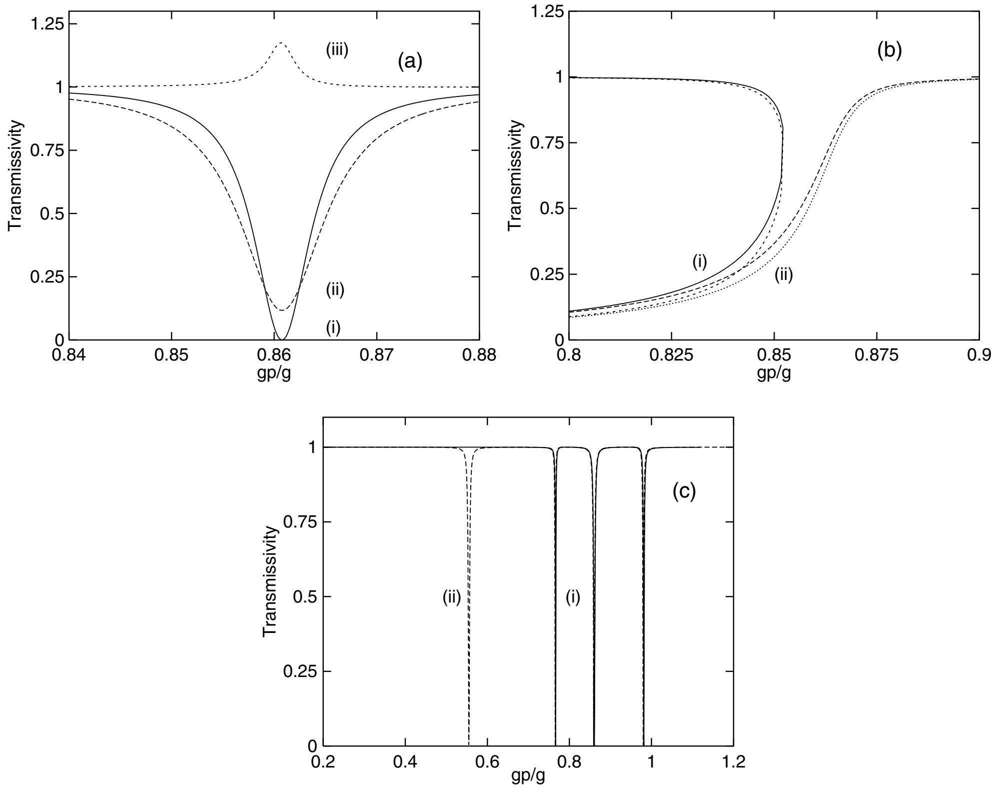

Figure 2. Plot of the transmission coefficient versus $gp/g = \gamma'/\gamma$ for (a) an off-channel site formed from linear dielectric media. Curves are for $a\epsilon = 0$ (i), $a\epsilon = 0.002$ (ii), and $a\epsilon = -0.002$ (iii). (b) As in (a) but with a Kerr nonlinearity of the form $\lambda|t|^2 = 0.000\,25$. Curves are for $a\epsilon = 0$ (i) and $a\epsilon = 0.0005$ (ii). (c) An off-channel cluster of three sites. The plots are of the $\epsilon = 0$, $\lambda = 0$ system (i) and the $\epsilon = 0$, $\lambda|E_{0,p+2}|^2 = 0.001$ Kerr system (ii). The curves labelled (i) and (ii) cannot be distinguished at the upper three resonances. For all plots in figures 2(a)–(c) the values of the constants a, b, c and k are the same.

The three curves in figure 2(a) all show a rapid variation in the transmission intensity with small changes in γ' for γ' in the vicinity of the $\gamma'/\gamma = 0.861$ resonance. This variation with γ' can be used to modulate the energy transmission in the waveguide channel. If γ' were changed by mechanical (piezoelectric), chemical, or electrical means, such changes would amplitude modulate the flow of optical energy in the waveguide. In the case of curve (iii) in figure 2(a) the amplitude modulation is further assisted by the amplification of the impurity medium.

We next consider the effects of nonlinearity in the off-channel site on the waveguide transmission [12]. Results are obtained from equation (7) and the third order equation for s. In the case where $[(\gamma c)^2\lambda t^2\gamma' a + 4(\gamma' a - 1)^3/27]/[\lambda t^2\gamma' a] > 0$, s has only one solution and the transmission coefficient is single valued. If this condition is not satisfied s has three solutions and the system exhibits a multiple valued transmission coefficient. Such systems with multiple valued transmission properties are said to be optically bistable. In figure 2(b) results are shown for an off-channel site with a positive Kerr nonlinearity (i.e., $\lambda > 0$). Plots are presented for the cases $\epsilon = 0$ and $\epsilon > 0$ (dissipation). The constants a, b, c, and k are chosen as in figure 2(a) and for the nonlinearity $\lambda|t|^2 = 0.000\,25$. (The results of our analytical theory for $\epsilon = 0$ are in qualitative agreement with recent weak coupling scattering theory studies of similar nonlinear off-channel sites [12].) The condition for a bound state at an isolated nonlinear single-site impurity is $\gamma' a(1 + \lambda t^2|s|^2) = 1$ [7, 10] so that for real solutions $s = [(\frac{1}{\gamma' a} - 1)/(\lambda t^2)]^{1/2}$. The dependence of the bound state on t and its renormalization by the coupling γc results in the optical bistability observed in the transmission coefficient below $gp/g = 0.85$. Resonant waveguide transmission for small ϵ is expected when $\gamma' a(1 + \lambda t^2|s|^2) \approx 1$. This is observed in the $\epsilon \neq 0$ results.

The solutions for the single-site transmission for $\epsilon = 0$ have some interesting symmetries. The equations are invariant under $s \to -s$, $c \to -c$. They are also invariant under $s \to -s$, $\gamma' a \to -\gamma' a + 2$, $\lambda \to -\frac{\gamma' a}{-\gamma' a+2}\lambda$. These relations allow for $\lambda < 0$ solutions to be written in terms of $\lambda > 0$ solutions. The $\lambda < 0$ solutions then also exhibit regions of optical bistability. In the strong coupling limit ($c \to \pm\infty$) we find $s \to [-\gamma c/\lambda t^2 \gamma' a]^{1/3}$ and $T \to 4\sin^2 k/[\gamma c^2(1 + 2\frac{b}{a}\sin k)]^2$ with no optical bistability.

The dielectric constant of the off-channel Kerr nonlinear site can be used to modulate the energy transmitted in the waveguide. For example, a pass band frequency pulse incident on the Kerr medium (even though the pass band frequency differs from the frequency of the energy in the waveguide) changes the dielectric properties, at the frequency of the waveguide modes, of the Kerr site. This results in modulation of the energy transported in the waveguide. Modulational effects can also be achieved by placing the Kerr impurity in a second waveguide and modulating the Kerr dielectric with pulses in the second waveguide.

3.2. Finite cluster of off-channel sites

Equations (3)–(6) can be generalized to treat a finite cluster of off-channel sites [3, 6, 16]. Consider a linear array of m off-channel sites composed of a Kerr dielectric medium for the case $\epsilon = 0$. The m off-channel sites (along the positive y-axis) lie on a line that is perpendicular to the waveguide channel (along the x-axis). This system is described by equations (3) and (4) with $f_0(E) = \gamma'[1 + \lambda|E|^2]E$, and

$$E_{0,p} = af_0(E_{0,p}) + bf_0(E_{0,p+1}) + \gamma c E_{0,0}, \tag{8}$$

$$E_{0,p+r} = af_0(E_{0,p+r}) + b(f_0(E_{0,p+r-1}) + f_0(E_{0,p+r+1})) \tag{9}$$

where $p > 0$, $1 \leqslant r \leqslant m - 2$, and

$$E_{0,p+m-1} = af_0(E_{0,p+m-1}) + bf_0(E_{0,p+m-2}). \tag{10}$$

The solutions of equations (3), (4), (8), (9), and (10) for the $\lambda = 0$ (linear medium case) give the transmission coefficient

$$T = \frac{4\sin^2 k}{4\sin^2 k + 4c\sin k\,\mathrm{Im}(\gamma' s)/(\gamma b) + c^2|\gamma' s|^2/(\gamma b)^2}. \tag{11}$$

The waveguide dispersion relation is again from the solution of $1 = \gamma[a(\omega) + 2b(\omega)\cos k]$, and $s = \frac{\gamma c}{\gamma' b}\frac{y_-^{m+1}y_+ - y_+^{m+1}y_-}{y_-^{m+1} - y_+^{m+1}}$ where $y_\pm = \alpha \pm [\alpha^2 - 1]^{1/2}$ and $\alpha = \frac{1-\gamma' a}{2\gamma' b}$. Using the parameter values in figure 2(a), waveguide transmission results are shown in figure 2(c) for a cluster of three off-channel sites. In this case a multiplicity of three resonances, arising from various bound states of the cluster, is observed at $gp/g = 0.767, 0.861$, and 0.981.

It is interesting to compare the result in equation (11) for an off-channel cluster of sites formed of linear dielectric media characterized by γ', $\epsilon = 0$, and $\lambda = 0$ with results for an off-channel cluster of sites formed of Kerr dielectric media characterized by γ', $\epsilon = 0$, and $\lambda \neq 0$ at each site. In figure 2(c) waveguide transmission results are also shown for the system of a cluster of three off-channel sites in the case in which a small Kerr nonlinearity is added. For these plots we have taken $\lambda|E_{0,p+2}|^2 = 0.001$. Three resonances are found at

Table 1. Normalized field intensities $\frac{|E_0|^2}{|E_{\max}|^2}$, $\frac{|E_1|^2}{|E_{\max}|^2}$, and $\frac{|E_2|^2}{|E_{\max}|^2}$ on the three nonlinear sites at each of the four different gp/g transmission resonances. Here 0 labels the site closest to the waveguide, 2 labels the site furthest from the waveguide, and $|E_{\max}|^2$ is the maximum intensity of the ILM resonance at $gp/g = 0.555$. This occurs on the 0 site.

$\frac{gp}{g}$	$\|E_0\|^2/\|E_{\max}\|^2$	$\|E_1\|^2/\|E_{\max}\|^2$	$\|E_2\|^2/\|E_{\max}\|^2$
0.555	1.0	0.070 6	0.001 90
0.766	0.001 87	0.003 81	0.001 90
0.859	0.001 89	0.000 00	0.001 90
0.980	0.001 90	0.003 76	0.001 90

$gp/g = 0.766$, 0.859, and 0.980. In addition, a fourth resonant condition at $gp/g = 0.555$ is observed. The $gp/g = 0.555$ resonance arises from the excitation of an odd parity intrinsic localized mode (ILM) [7–11]. (The ILM is a soliton-like excitation [10]. A detailed discussion can be found in [10].) The maximum of the ILM lies closest to the waveguide channel and the ILM decays away from the waveguide channel in a way characteristic of soliton pulses. In table 1 the field intensity squared (i.e., $|E|^2$) of the $\lambda \neq 0$ results is presented in each of the three Kerr sites for the four different gp/g resonances. The intensity is normalized to the maximum intensity of the ILM mode at $gp/g = 0.555$. The fields in the ILM mode are characteristically much more intense than are those of the modes in non-ILM resonances.

4. Semi-infinite impurities

An interesting generalization of the single off-channel site problem is made by replacing the off-channel impurity site by an impurity site that is part of a semi-infinite waveguide (see figure 1(b)). The original infinite waveguide of linear dielectric media is still taken to be along the x-axis at $(n, 0)$ for n over the integers. The impurity site (of Kerr dielectric media) is located at $(0, p)$ for $p > 0$ and the semi-infinite waveguide channel (formed of the same linear dielectric media as the infinite waveguide) is located at $(0, m)$ where $m \geqslant (p + 1)$.

The set of difference equations for this system is equations (3), (4), and (6) for $\epsilon = 0$ with the addition, in place of equation (5), of [6]

$$E_{0,p} = af_0(E_{0,p}) + \gamma c E_{0,0} + \gamma b E_{0,p+1}, \tag{12}$$

$$E_{0,p+1} = \gamma[aE_{0,p+1} + bE_{0,p+2}] + bf_0(E_{0,p}), \tag{13}$$

and

$$E_{0,m} = \gamma[aE_{0,m} + b(E_{0,m-1} + E_{0,m+1})] \tag{14}$$

for $m \geqslant p + 2$. Equation (12) describes an impurity located at the end of a semi-infinite waveguide channel. The semi-infinite waveguide channel is described by equations (13) and (14). Equations (3) and (4) describe the infinite waveguide. The scattering geometry is the same as in section 3, i.e., incident and reflected waves in $(n, 0)$ with $n < 0$.

The transmission coefficient for scattering into the infinite waveguide in the $\epsilon = 0$ case is given by

$$T = \frac{4\sin^2 k}{|2\sin k - \mathrm{i}\frac{\gamma' c}{\gamma b}(1 + \lambda t^2|s|^2)s|^2}. \tag{15}$$

Here the dispersion relation in the infinite and semi-infinite waveguide channels is given as a solution of $1 = \gamma[a(\omega) + 2b(\omega)\cos k]$, k is the wavenumber of the waveguide mode, γ'

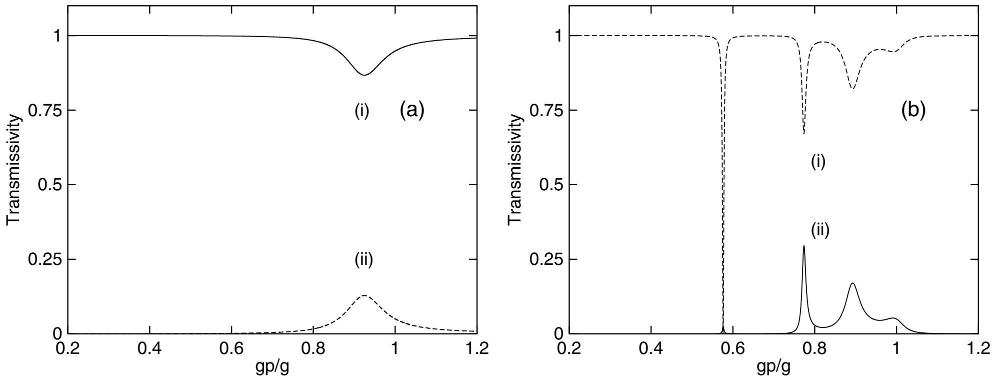

Figure 3. Plot of the transmission coefficients versus $gp/g = \gamma'/\gamma$ for (a) a single-site impurity with γ', $\epsilon = 0$, and $\lambda = 0$ at the end of a semi-infinite waveguide channel. The curve labelled (i) is for transmission into the infinite waveguide; the curve labelled (ii) is for transmission into the semi-infinite waveguide. (b) Three Kerr sites with γ', $\epsilon = 0$, and $\lambda|E_{0,p+2}|^2 = 0.001$ at the end of a semi-infinite waveguide channel. The curve labelled (i) is for transmission into the infinite waveguide; the curve labelled (ii) is for transmission into the semi-infinite channel. For all plots here and in figure 2 the same values of a, b, c, and k are used and the parameters in figures 3 characterizing the infinite and semi-infinite waveguides are the same.

characterizes the impurity site at $(0, p)$, and s is a solution of $\gamma'[a(\omega) + b(\omega)e^{ik}](\lambda t^2|s|^2 + 1)s - s + \gamma c = 0$. The transmission coefficient for scattering into the semi-infinite waveguide is given by

$$T_{\text{semi}} = \left|\frac{\gamma'}{\gamma}(1 + \lambda t^2|s|^2)s\right|^2 T, \tag{16}$$

and the reflection coefficient is obtained from the relation $R + T + T_{\text{semi}} = 1$. In figure 3(a) results for T and T_{semi} are shown as a function of $gp/g = \gamma'/\gamma$ for $\lambda = 0$ (case of linear media) and γ, a, b, and k used in figure 2.

A generalization can be made to the case in which the single nonlinear site at the end of the the semi-infinite waveguide is replaced by a cluster of three nonlinear sites similar to those in section 3.2. Results for this system are shown in figure 3(b). A transmission anomaly associated with an ILM generated in the three Kerr sites is observed at $gp/g = 0.577$.

4.1. Off-channel 'T' junctions

Another type of circuit element that a waveguide can interact with is an off-channel junction between three semi-infinite waveguides [6, 7, 10, 11] (see figure 1(c)). Consider an infinite waveguide with a channel along the x-axis characterized by parameters γ, a, and b. The off-channel junction between the semi-infinite waveguides forms a 'T' with the top of the 'T' parallel to the infinite waveguide on the x-axis and the stem of the 'T' on the y-axis directed away for the infinite waveguide. The channels of the three semi-infinite waveguides are characterized by the same parameters (γ, a, and b) as those of the infinite waveguide. For simplicity, a coupling is taken between the single site at the junction and its closest neighbour on the infinite waveguide. The dielectric properties of the junction site are characterized by the parameters γ', $\lambda = 0$ and the coupling between the junction site and the infinite waveguide is characterized by c.

The transmission in this system for radiation in the infinite waveguide that is incident on the coupling to the junction can be computed exactly. The transmission coefficient for incident radiation scattered into the infinite waveguide channel is

$$T = \frac{[1-\gamma'(a+3b\cos k)]^2 + 9[\gamma' b \sin k]^2}{[1-\gamma'(a+3b\cos k)]^2 + 9[\gamma' b \sin k(1+\frac{c^2}{6b^2\sin^2 k})]^2} \tag{17}$$

and the transmission coefficient for scattering into one of the semi-infinite waveguide channels is

$$T_{\mathrm{semi}} = \frac{(\gamma' c)^2}{[1-\gamma'(a+3b\cos k)]^2 + 9[\gamma' b \sin k(1+\frac{c^2}{6b^2\sin^2 k})]^2}. \tag{18}$$

In equations (17) and (18) k is the wavenumber of the radiation propagating in the waveguides, and the dispersion relation in each waveguide is given by the solution of $1 = \gamma[a(\omega) + 2b(\omega)\cos k]$. The reflection coefficient, R, in the absence of dissipation or amplification is obtained from the relation $1 = T + 3T_{\mathrm{semi}} + R$.

If the dielectric media of the three semi-infinite waveguides were changed so that they no longer supported propagating modes but supported a mode bound to the junction [7], resonant scattering in the infinite waveguide could be observed due to resonant interaction with the junction bound state mode. In this case the dielectric parameter of the semi-infinite waveguide channels, γ_{semi}, satisfies $1 = \gamma_{\mathrm{semi}}[a(\omega) + 2b(\omega)\cosh q]$ where q characterizes the exponent decay, $\exp -qn$, of the junction bound state in the semi-infinite waveguide channel sites. (Here we label the channel sites in the semi-infinite channel by integers n leading away from the junction site.) The energy transmitted in the infinite waveguide channel is given by $T = \frac{[1-\gamma'(a+3b\mathrm{e}^{-q})]^2}{[1-\gamma'(a+3b\mathrm{e}^{-q})]^2+[\gamma' c \frac{c}{2b\sin k}]^2}$, where the notation here follows that in equations (17) and (18). A transmission resonance is exhibited at the bound state condition $1 = \gamma_{\mathrm{semi}}(a(\omega) + 3b(\omega)\mathrm{e}^{-q})$ [7]. This is an example of how elements in adjacent circuits may not support the propagation of energy but can still influence the propagation of energy in an infinite propagating waveguide channel.

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